Novel Modal Decomposition Methods of Fluid Flow

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Introduction

Model order reduction is a key enabler for flow control and better understanding of the flow physics. The most of reduction techniques, like Galerkin method, base on the approximation of the solution by superposition of global modes. In present paper, two novel modal decompositions of fluid flow are proposed.

Recursive DMD

The first one is data-driven and is comprising key features of proper orthogonal decomposition (POD) [2] and dynamic mode decomposition (DMD)[1]. The first mode is the normalized real or imaginary part of the DMD mode that minimizes the time-averaged residual. The Nth mode is defined recursively in an analogous manner based on the residual of an expansion using the first N - 1 modes. The resulting recursive DMD (RDMD) modes [3] are orthogonal by construction, retain pure frequency content and aim at low residual. Recursive DMD is applied to transient cylinder wake data and is benchmarked against POD and DMD for the same snapshot sequence (fig. 1). Unlike POD modes, RDMD structures are shown to have purer frequency content while retaining a residual of comparable order to POD. In contrast to DMD, with exponentially growing or decaying oscillatory amplitudes, RDMD clearly identifies initial, maximum and final fluctuation levels. Intriguingly, RDMD outperforms both POD and DMD in the limit-cycle resolution from the same snapshots. Robustness of these observations is demonstrated for other parameters of the cylinder wake and for a more complex wake behind three rotating cylinders. Recursive DMD is proposed as an attractive alternative to POD and DMD for empirical Galerkin models, in particular for nonlinear transient dynamics.

Complex Dynamic Response

The second approach, called Complex Dynamic Response [4], allows the computation of global, physical modes. Presented approach, in contrast to the empirical methods such as POD and DMD, does not require prior solution of unsteady governing equations.

The CDR modes (fig. 2) are computed in frequency domain as a response of the flow (disturbance equation) to random or localized perturbation. It allows continuous selection of the desired structures, for a given values of frequency and growth rate (by the choice of complex shift λ), as well as the position of perturbation. As a result, modes for slightly varying operating conditions might be computed by simple change of a few parameters in the simulation.

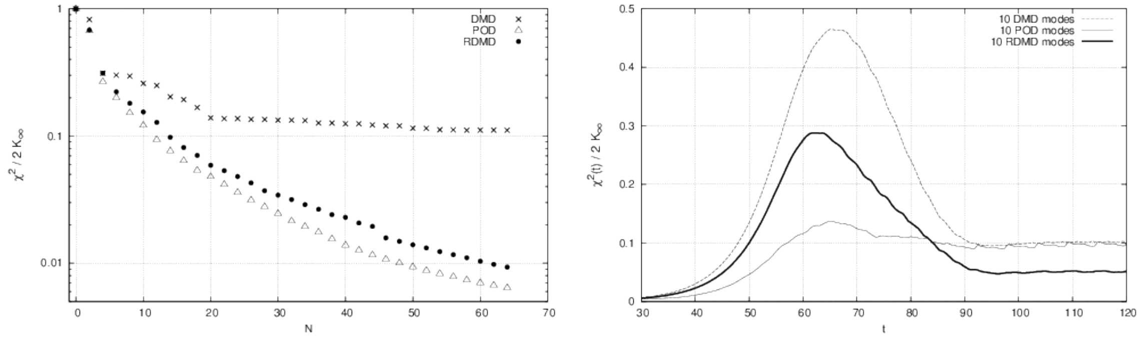

Figure 1: The time-averaged fluctuation level of the residual with increasing number of modes (left) and instantaneous truncation error (right) for POD, DMD and RDMD. The value is normalized by the corresponding fluctuation level (2K∞) on the limit cycle.

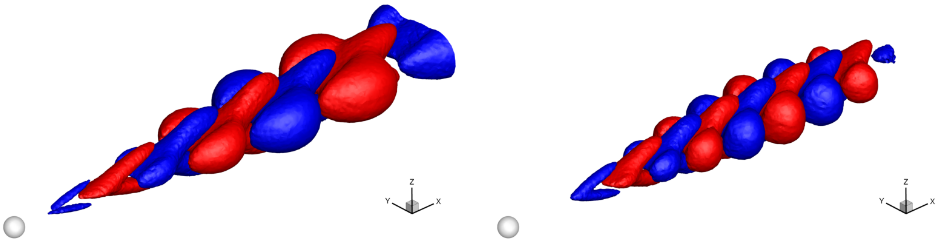

Figure 2: Iso-surfaces of transverse velocity for real parts of CDR modes obtained for different complex shifts. Left: $\lambda_{Im} = 0.7$, right: $\lambda_{Im} = 1.4$

References

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