Application of the POD method to optimal design of experiment

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Why should we invest in a design of an experiment

- Number of probe locations is always limited.
- For more complex problems intuition is not enough.
- To obtain maximum amount of information from an experiment.
- To reduce measurement error.

Method outline

1. Perform N numerical experiments which cover some parameter space (e. g.: different angles of attack, inlet velocities).
2. Create reduced linear model based on numerical data.
3. Use reduced model to construct a statistical criterion.
4. Choose measurement points that will minimize the variance of the model parameters.
5. Perform the experiment and measure the flow parameters in optimal positions.
6. Reconstruct the flow field by using the reduced model and the experiment data.

Proper Orthogonal Decomposition I

CFD Simulation $\rightarrow$ Reduced model $\rightarrow$ optimization

measurement + Reduced model $\rightarrow$ reconstruction

Requirements:

- Generic.
- Numerically stable.

Reduced model by using the Proper Orthogonal Decomposition:

$$\mathbf{V} = [\mathbf{V_1}, \ldots, \mathbf{V_n}] \approx \left[\sum_{i=0}^{m} \mathbf{M_i} \cdot \beta_{1i}, \ldots, \sum_{i=0}^{m} \mathbf{M_i} \cdot \beta_{ni} \right]$$

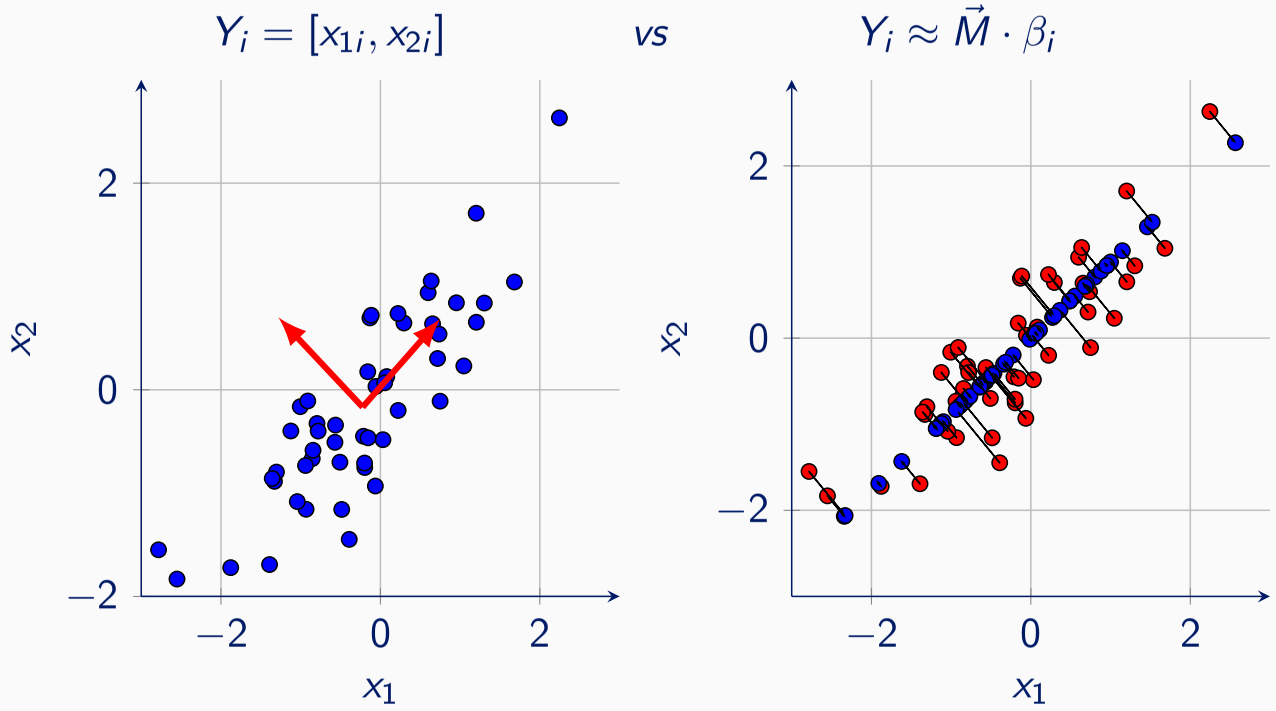
$\mathbf{V_i}$ - Data in column vector

$\mathbf{M_i}$ - Basis vectors obtained by the POD

$\beta_{i.}$ - Model coefficients vector

Proper Orthogonal Decomposition II

$Y_i = [x_{1i}, x_{2i}]$ *vs* $Y_i \approx \vec{M} \cdot \beta_i$



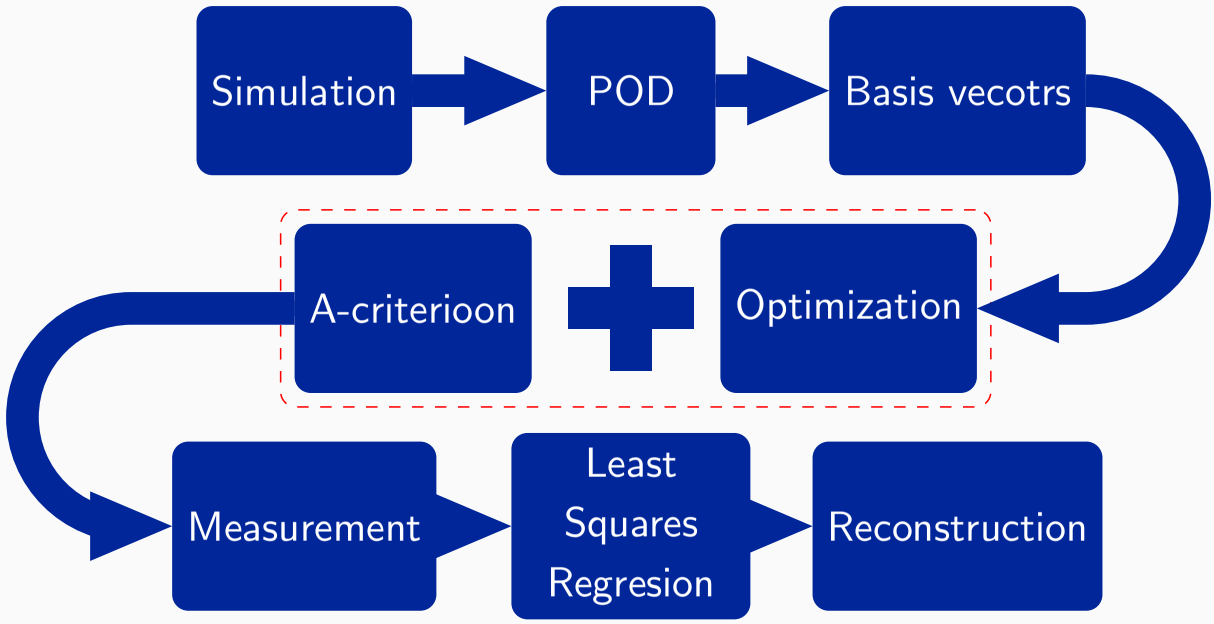
Statistical criterion

- Linear model describing measured phenomenon:

$$\mathbf{Y} = \mathbf{X} \cdot \beta + \varepsilon$$

  Y - measurement data at any position that can be predicted by the model, β - model coefficients, **X** - model constants (matrix constructed from POD basis), ε - random error.

- Only few equations are required to find β. Selecting an appropriate relations can improve accuracy of the β estimation.
- A-criterion - statical tool allowing to measure variance of estimated parameter β with respect to selected equations (corresponding to measurement positions).

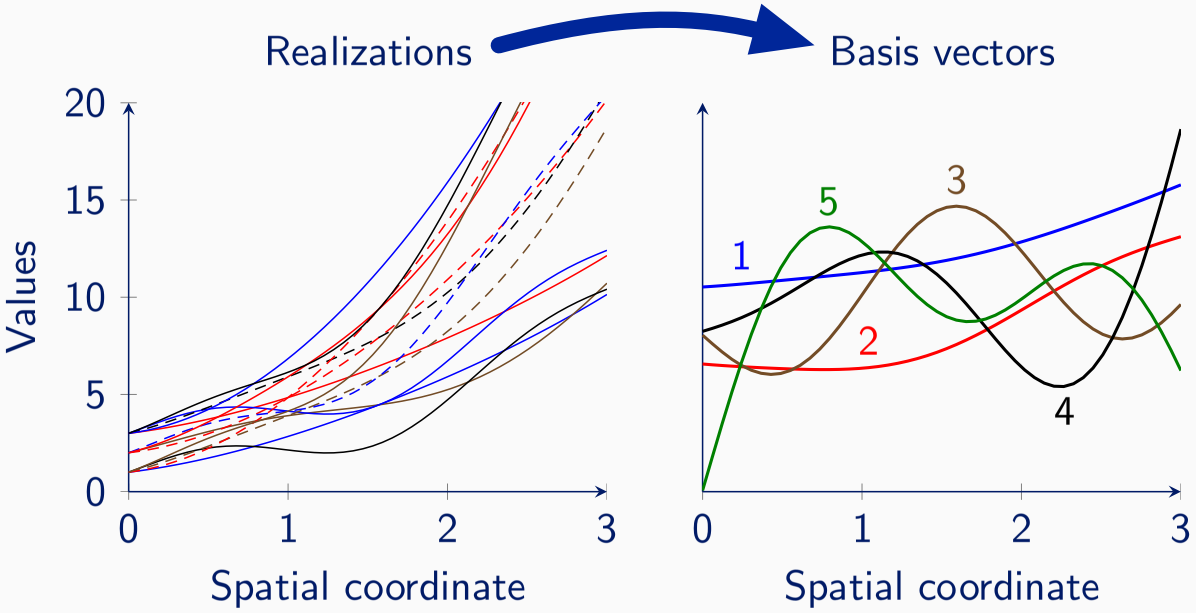


Exemplary case

Exemplary case I

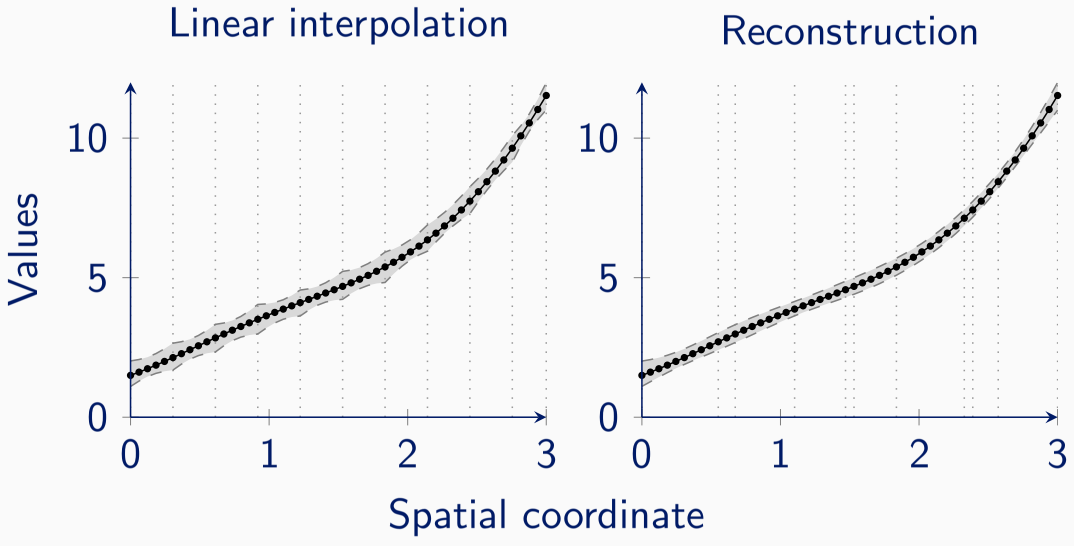
$$y = a \cdot x^2 + sin(b \cdot x) + c$$

$$a, b, c = \{1, 2, 3\}$$



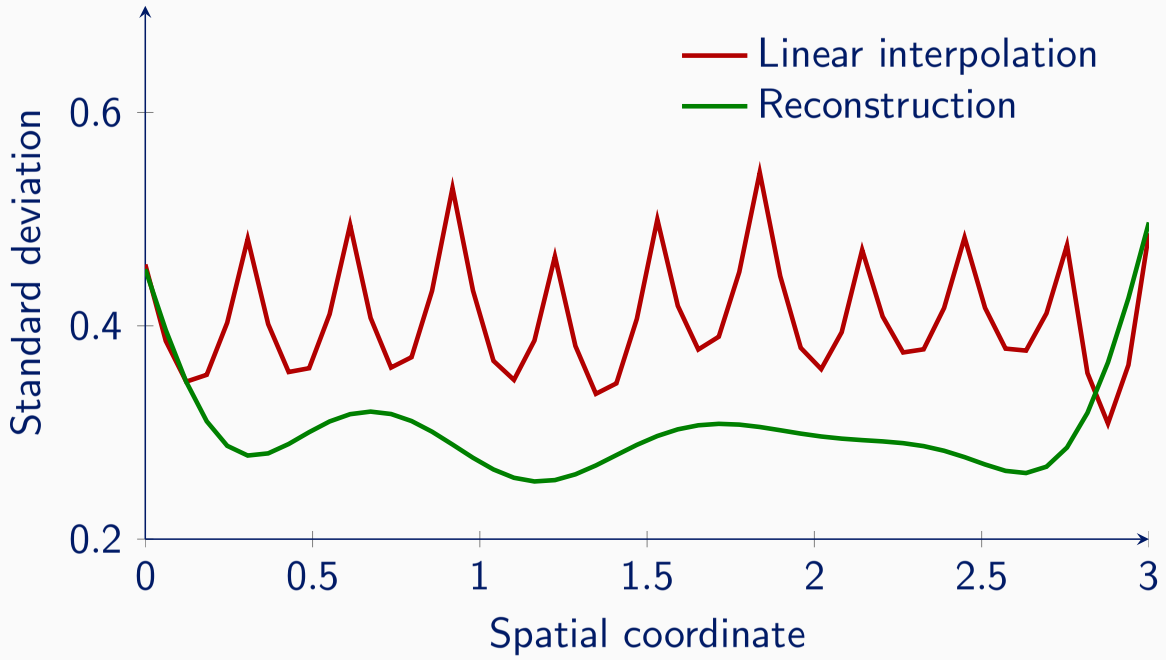
Exemplary case II

$a = 1.2$ $b = 1.8$ $c = 1.5$



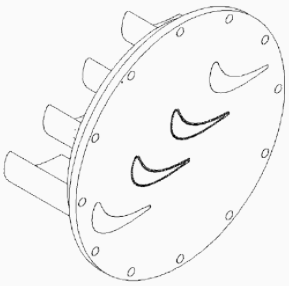
Measurement pos. σ Exact

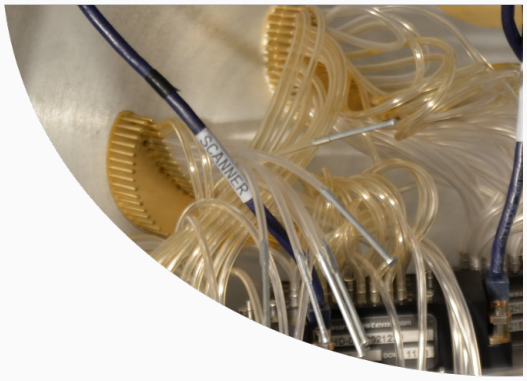
$a = 1.2$ $b = 1.8$ $c = 1.5$



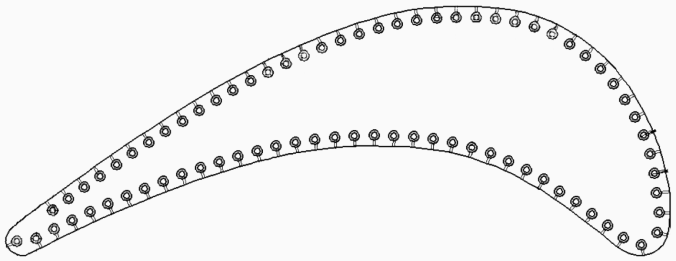
Method validation basing on the experimental data

Experiment





Experiment - 69 measurement positions on the blade

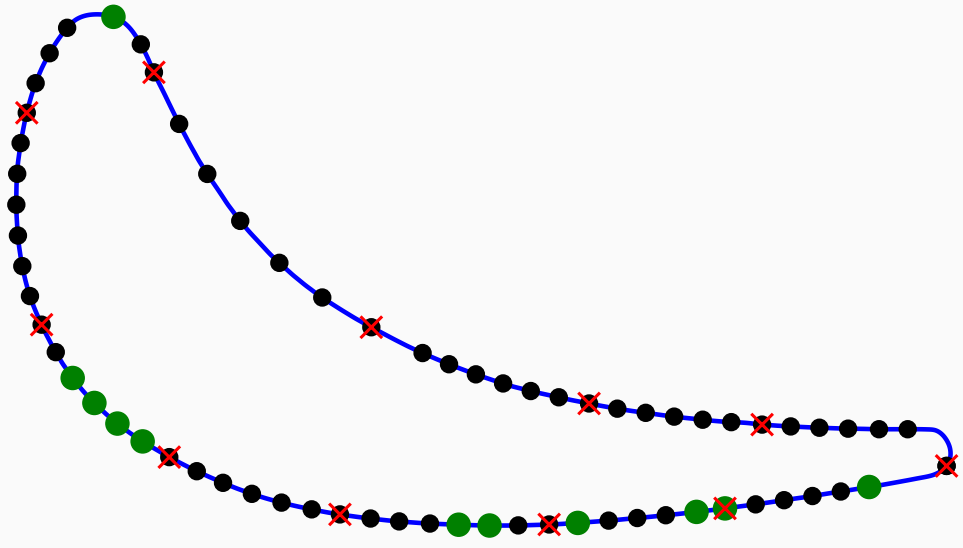


Optimization

- Choose 11 optimal out of 69 available positions
- Reduced model constructed(5 basis vectors and parameters) from 200 flow configurations
- Input data for reduced model include variation in angle of attack and inlet Mach number.

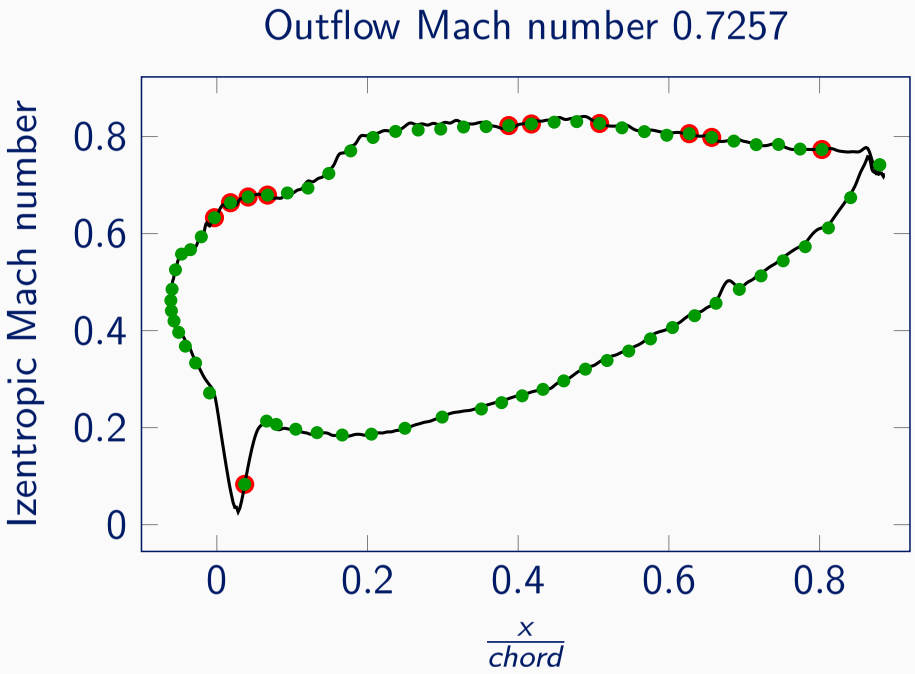
Subsonic case

Optimized measurement positions

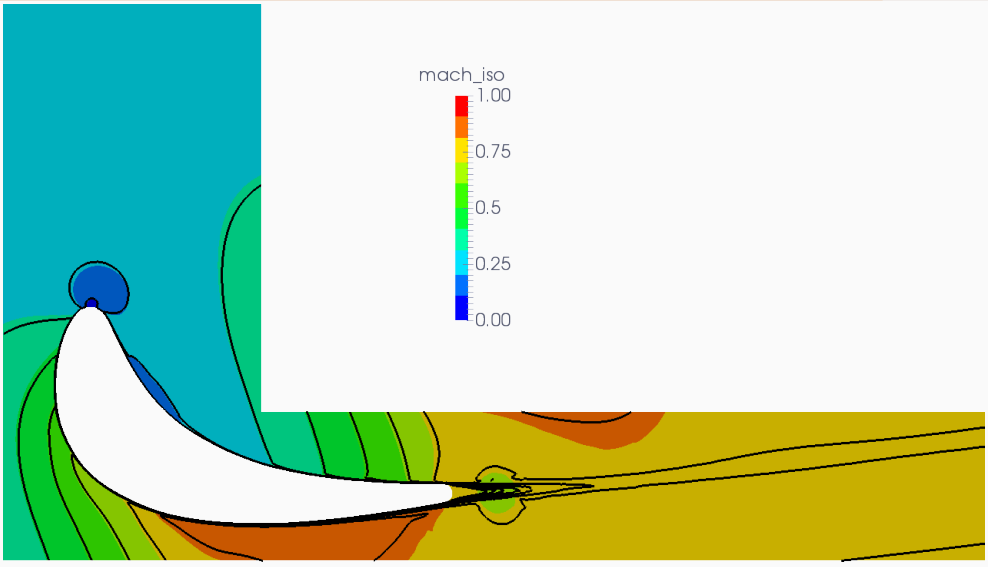


Blade profile
Accesible positions
Optimal positions
Equal, initial distribution

Reconstruction of the Mach number on the blade



Flow field reconstruction

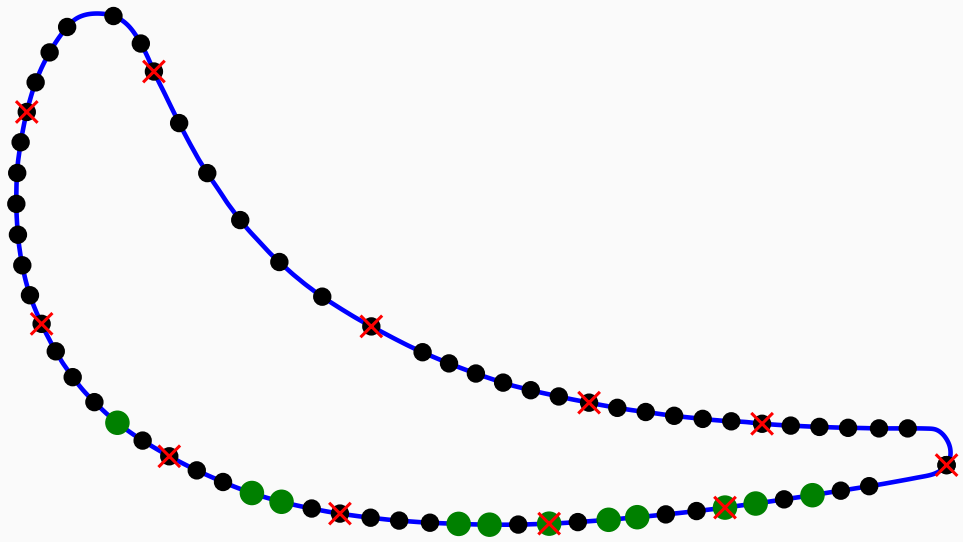


Outflow mach number 0.737

- Filled color - reconstruction
- Black contour lines - closest CFD solution to the experimental conditions

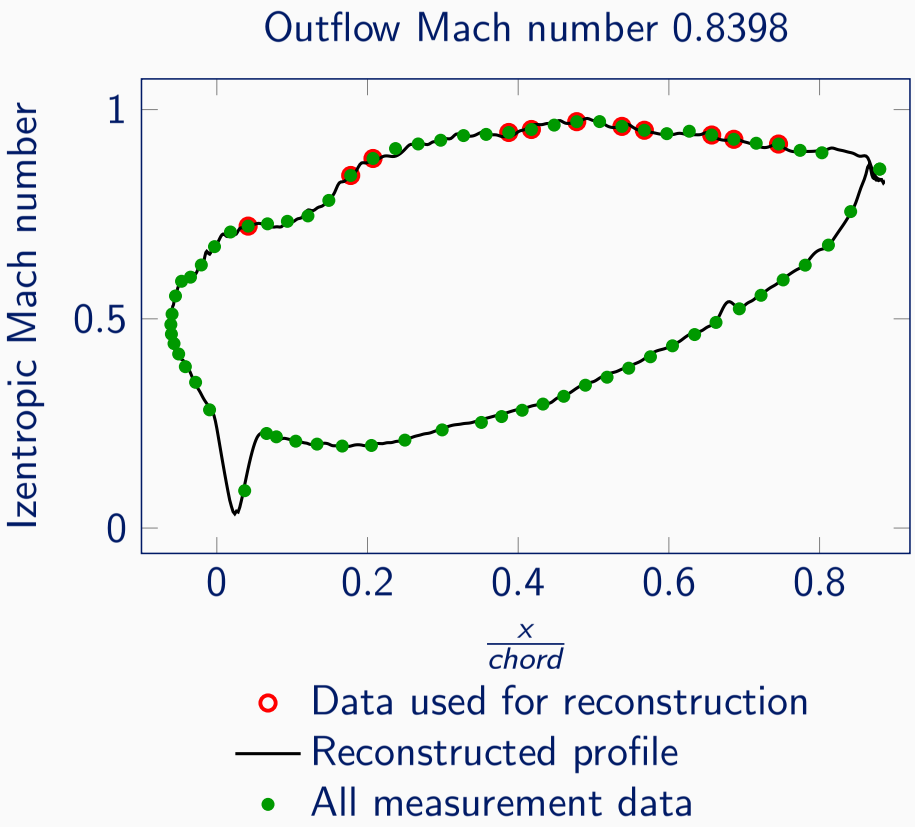
Sub and supersonic case

Optimized measurement positions

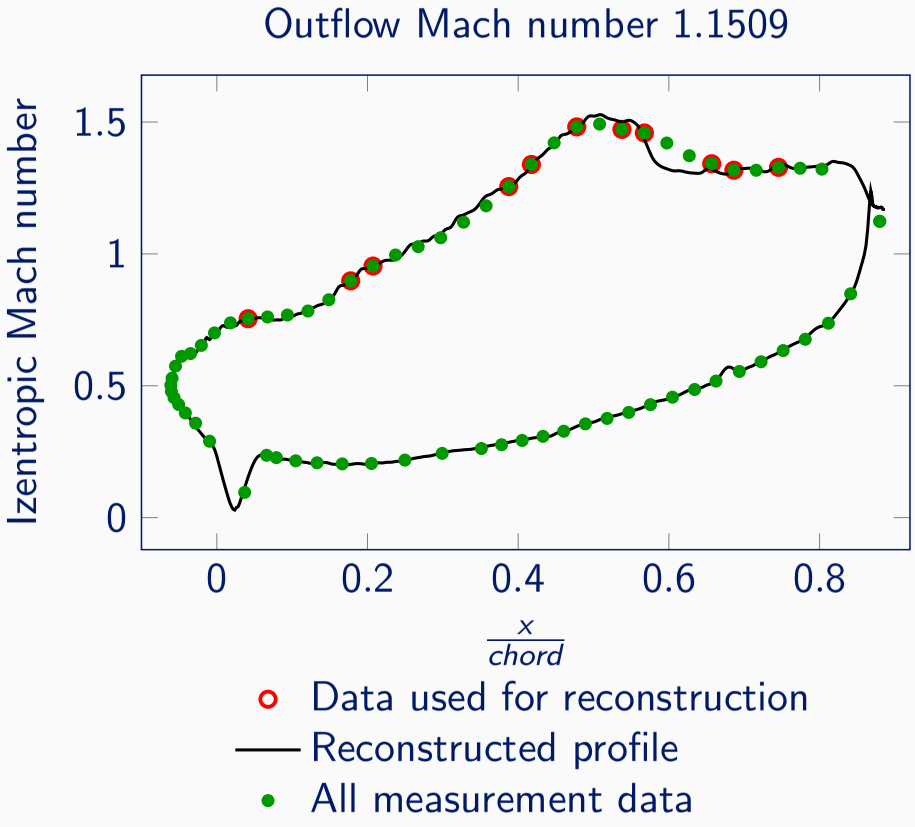


Blade profile
Accessible positions
Optimal positions
Equal, initial distribution

Isentropic Mach number reconstruction on the blade



Isentropic Mach number reconstruction on the blade



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Thank you for your attention