Novel approaches to estimating turbulent EDR from low and moderate resolution velocity time series

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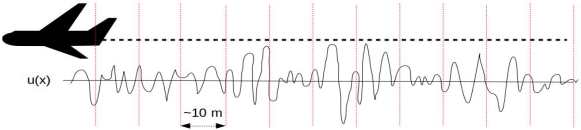
Experiments in Fluid Mechanics, Warsaw, 23-24.10.2017

Motivation

- Information on turbulent kinetic EDR based on in-situ airborne measurements is still scarce.
- Research aircraft are often not equipped to measure wind fluctuations with spatial resolution better than few tens of meters
- Estimates of ϵ at such low resolutions using power spectral density or structure functions are complex and far from being standardised



Motivation

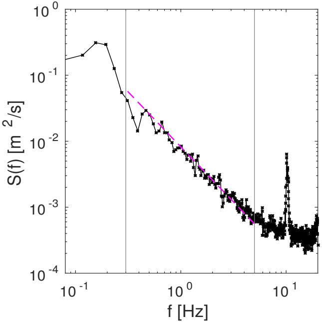
Can we introduce alternative methods to increase robustness of ϵ retrievals?

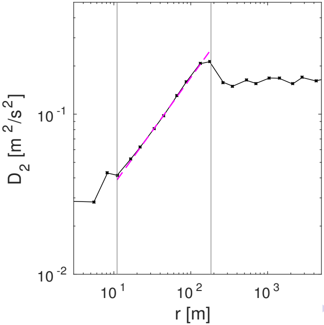
Kopeć, J.M., Kwiatkowski, K., de Haan, S. and Malinowski, S.P.: Retrieving atmospheric turbulence information from regular commercial aircraft using Mode-S and ADS-B, Atmos. Meas. Tech., **9**, 2016

Existing methods for ϵ retrieval

Inertial-range scaling of power spectral density and 2^{nd}-order structure function $C_1 \approx 0.49$, $C_2 \approx 2$.

$$S(f) = C_1 \left(\frac{U}{2\pi}\right)^{2/3} \epsilon^{2/3} f^{-5/3}, \quad D_2(r) = C_2 \epsilon^{2/3} r^{2/3}.$$





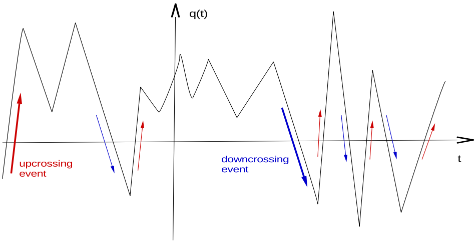
Existing methods for ϵ retrieval

Method based on the number of zero-crossings

Rice [Bell. Syst. Tech. J., **24**, 1945]

$$N^2 = \frac{\langle(\partial u'/\partial x)^2\rangle}{\pi^2\langle u'^2\rangle}.$$

N - number of zero-crossings per unit length



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$$\epsilon = 15\nu \left\langle \left(\frac{\partial u^{'}}{\partial x} \right)^2 \right\rangle = 15\nu \frac{\langle u'^2 \rangle}{\lambda_n^2} \quad \lambda_n \text{ -Taylor microscale}$$

Sreenivasan et al. [J. Fluid Mech., **137**, 1983]

$$N^2 = \frac{\langle (\partial u' / \partial x)^2 \rangle}{\pi^2 \langle u'^2 \rangle} \implies \epsilon = 15\pi^2 \nu u'^2 N^2$$

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$$N^2 = \frac{\langle (\partial u' / \partial x)^2 \rangle}{\pi^2 \langle u'^2 \rangle} \quad \Longrightarrow \quad \epsilon = 15\pi^2 \nu u'^2 N^2$$

New proposals

Method based on the number of zero-crossings can be used for signals resolved down to the smallest dissipative eddies...

...which is not the case for airborne velocity measurements.

New proposal: Reformulation of the original zero-crossing method in order to estimate ϵ from N of signals with spectral cut-off.

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New proposals

N is related to the dissipation spectra $\sim 2\nu f^2 S(f)$:

$$u'^2 N^2 = 4\int_0^\infty f^2 S(f)\mathrm{d}f \quad \text{and} \quad u'^2_{cut} N^2_{cut} = 4\int_0^{f_{cut}} f^2 S(f)\mathrm{d}f$$

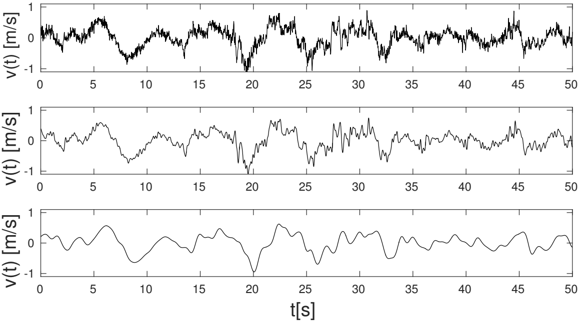
u'_{cut}, N_{cut} calculated for a signal with spectral cut-off at f_{cut}.
In the inertial range $S(f) = C_1(U/2\pi)^{2/3}\epsilon^{2/3}f^{-5/3}$. Subtracting equations for two different f_{cut}, $f_1 < f_2$ from the inertial range

$$(u_2'^2 N_2^2 - u_1'^2 N_1^2) = 3C_1\left(\frac{U}{2\pi}\right)^{2/3}\epsilon^{2/3}\left(f_2^{4/3} - f_1^{4/3}\right).$$

Wacławczyk M., Ma Y.-F., Kopeć J., Malinowski S. P. Novel approaches to estimating turbulent kinetic energy dissipation rate from low and moderate resolution velocity fluctuation time series, Atmos. Meas. Tech. Discuss., 2017

New proposals

Successive filtering of a signal with $f_1 < f_2 < ... < f_i < ....$



For each signal with cut-off f_i we calculate the number of zero-crossings per unit time N_i

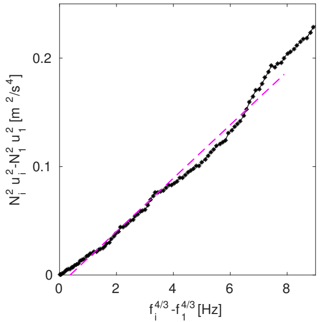
New proposals

$$\left(u_2'^2 N_2^2 - u_1'^2 N_1^2\right) = 3C_1 \left(\frac{U}{2\pi}\right)^{2/3} \epsilon^{2/3} \left(f_2^{4/3} - f_1^{4/3}\right).$$

With linear curve-fitting

$$3C_1 \left(\frac{U}{2\pi}\right)^{2/3} \epsilon^{2/3}$$

is estimated and the corresponding value of ϵ is found



New proposals

Second proposal: Recovering the missing part of the spectrum

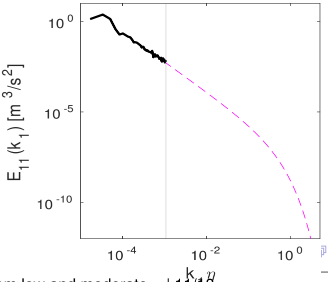
$$u'^2 N^2 = u'^2_{cut} N^2_{cut} \frac{\int_0^\infty k_1^2 E_{11} \mathrm{d}k_1}{\int_0^{k_{cut}} k_1^2 E_{11} \mathrm{d}k_1} = u'^2_{cut} N^2_{cut} \left(1 + \frac{\int_{k_{cut}}^\infty k_1^2 E_{11} \mathrm{d}k_1}{\int_0^{k_{cut}} k_1^2 E_{11} \mathrm{d}k_1}\right)$$

assume certain form of the energy spectrum
[Pope, *Turbulent flows*, 2001]

$$E(k) = C\epsilon^{2/3} k^{-5/3} f_\eta(\beta k \eta),$$

with ($\beta = 5.2$ and $c_\eta = 0.4$)

$$f_\eta = \mathrm{e}^{\left\{-\left[(\beta k\eta)^4 + (\beta c_\eta)^4\right]^{1/4} + \beta c_\eta\right\}}$$



New proposals

Next, use relation between $E(k)$ and $E_{11}(k_1)$, to finally arrive at

$$u'^2 N^2 \approx u'^2_{cut} N^2_{cut} \underbrace{\left[1 + \frac{\int_{k_{cut}\beta\eta}^{\infty} \xi_1^2 \int_{\xi_1}^{\infty} \xi^{-8/3} f_\eta(\xi) \left(1 - \frac{\xi_1^2}{\xi^2}\right) \mathrm{d}\xi \mathrm{d}\xi_1}{\int_0^{k_{cut}\beta\eta} \xi_1^2 \int_{\xi_1}^{\infty} \xi^{-8/3} f_\eta(\xi) \left(1 - \frac{\xi_1^2}{\xi^2}\right) \mathrm{d}\xi \mathrm{d}\xi_1} \right]}_{\mathcal{C}_{\mathcal{F}}}$$

$$u'^2 N^2 = u'^2_{cut} N^2_{cut} \mathcal{C}_{\mathcal{F}},$$

where $\mathcal{C}_{\mathcal{F}}$ is a correcting factor to be calculated

$$\epsilon = 15\pi^2 \nu u'^2_{cut} N^2_{cut} \mathcal{C}_{\mathcal{F}}$$

New proposals

The integral bounds contain the Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$

Iterative procedure:

- Calculate $u'^2_{cut} N^2_{cut}$ of a signal
- Guess first value ϵ^0, $\eta^0 = (\nu^3/\epsilon^0)^{1/4}$
- Calculate $\mathcal{C}_{\mathcal{F}}$
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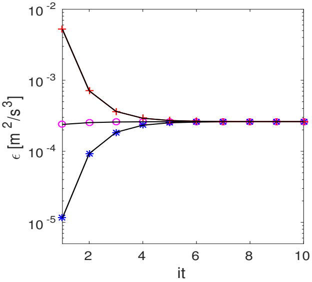
- Repeat the procedure



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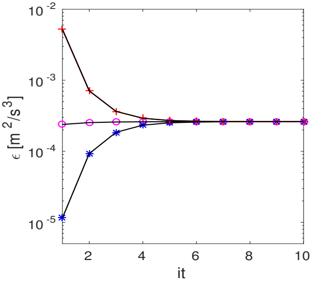

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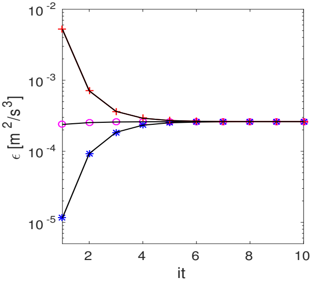

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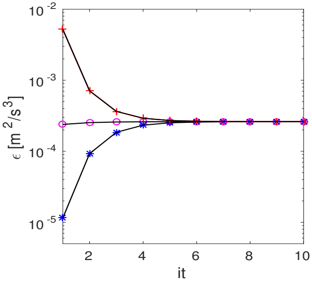

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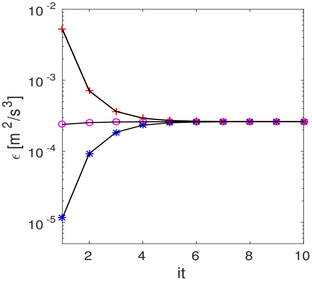
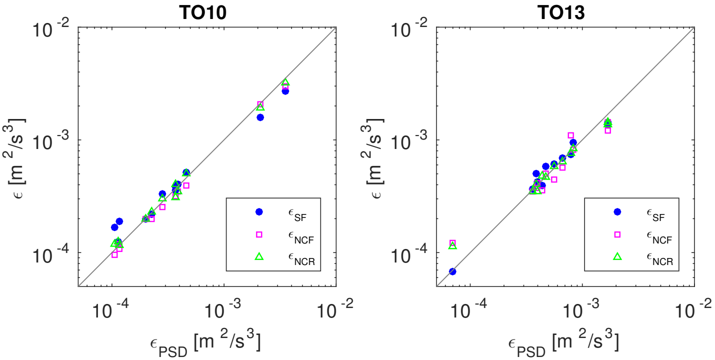


Results - POST measurements

POST research campaign flights *TO*10 and *TO*13

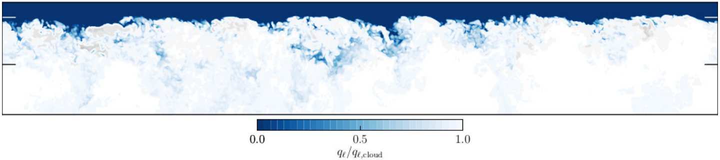
La Plante et al.: Physics of Stratocumulus Top (POST): turbulence characteristics, Atmos. Chem. Phys., **16**, 2016.



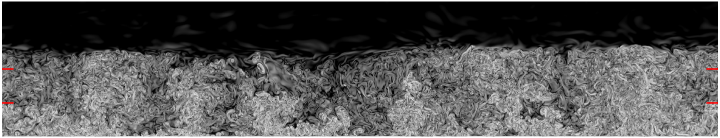
Results - DNS data

courtesy of Prof. J.-P. Mellado from the Max Planck Institute for Meteorology

stratocumulus cloud-top simulations



free convection



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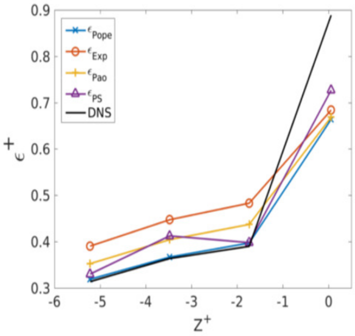

Figure: Stratocumulus cloud-top

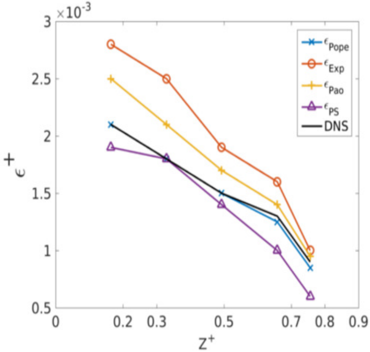

Figure: Free convection

Conclusions

- Two new methods for ϵ retrieval using number of crossings per unit length were proposed.
- Possible advantages are:
  - Increased roboustness of ϵ retrieval
  - New methods seem to be less sensitive to bias errors due to finite averaging windows
  - Second method - can be used for signals with large part of the dissipative spectrum resolved
- Disadvantages
  - Possibly larger scatter of results (at least in the case of artificial velocity fields)
  - Additional assumptions needed (Gaussianity of the process)

Acknowledgements

The financial support of the National Science Centre, Poland (project No. 2014/15/B/ST8/00180 and 2016/21/B/ST8/01010) is gratefully ackowledged. The POST field campaign was supported by US National Science Foundation through grant ATM-0735121 and by the Polish Ministry of Science and Higher Education through grant 186/W-POST/2008/0.

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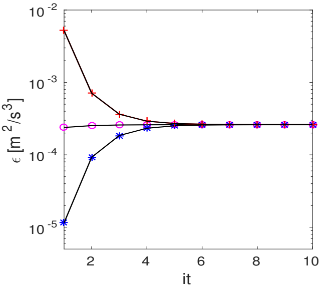

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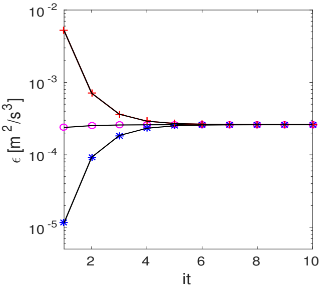

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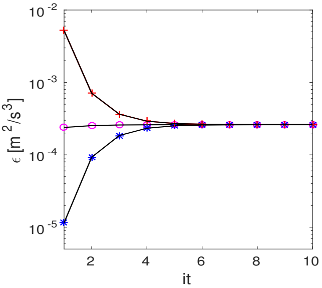

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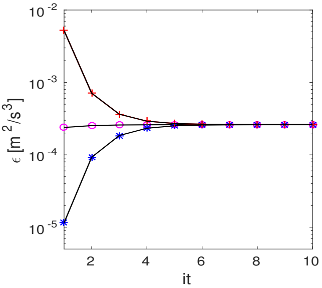

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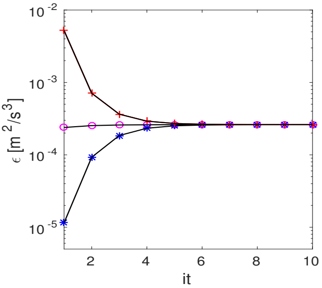

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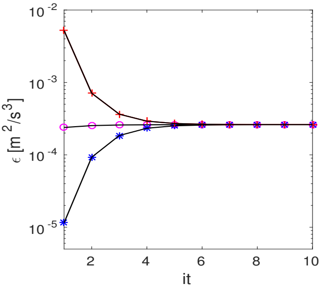

Results - artificial velocity fields

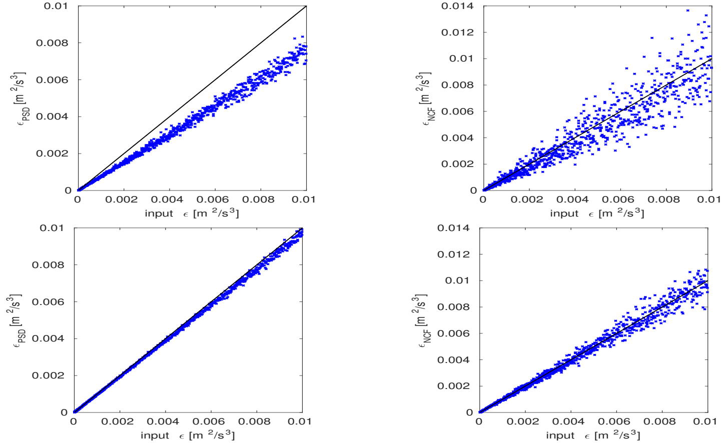

Figure: Estimated values of ϵ_{PSD} and ϵ_{NCF} for the 200 Hz synthetic signals and fitting range $1 - 20$ Hz as functions of corresponding input ϵ for upper plots: signals with $L \approx 50L_0$, lower plots: signals with $L \approx 400L_0$.