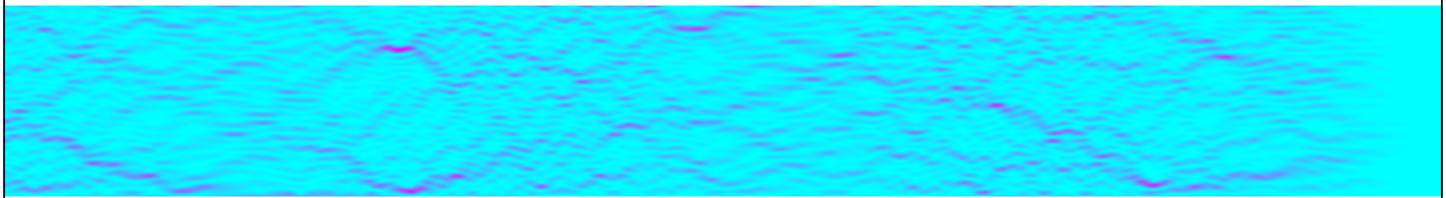
Waves over variable bathymetry – branched flow in the linear regime.

Adam Piotr Anglart[1,4]



in collaboration with T. Humbert[2], P. Petitjeans[1], V. Pagneux[2], A. Maurel[3]

[1]École Supérieure de Physique et de Chimie Industrielles de la ville de Paris,
Laboratoire de Physqiue et Mécanique des Millieux Hétérogènes, Paris, France

[2]Laboratoire d'Acoustique de l'Université du Maine, Le Mans, France

[3]Institut Langevin LOA, Paris, France

[4]Warsaw University of Technology,
The Faculty of Power and Aeronautical Engineering, Warsaw, Poland

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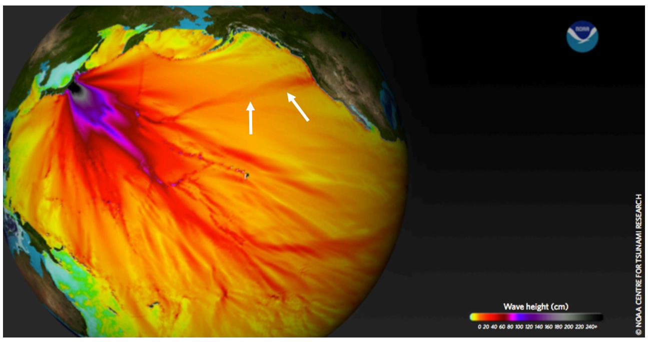
Surface water waves

High energy waves



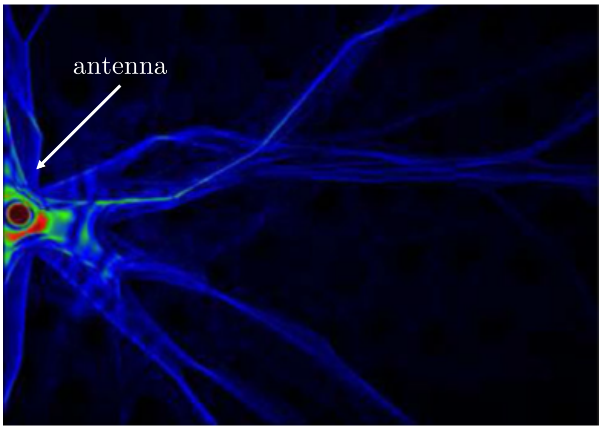
Branched flow seen in the wave energy map produced after the 2011 Sendai earthquake in Japan. High energy path heading for Crescent City in northern California.
National Oceanic and Atmospheric Administration (2011)

Intensity of a plane wave propagating from left to right in a random bathymetry
Degueldre *et al.* Nature Physics 12 (2016)

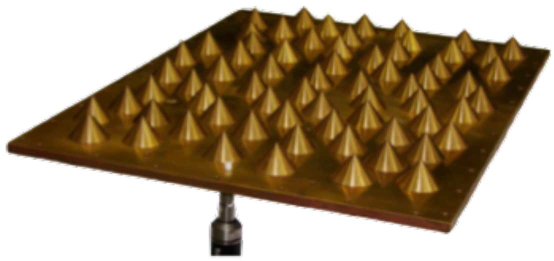
Shallow-water waves
very sensitive to small fluctuations
of the bottom topography

High energy paths

Microwaves experiment



Microwave pattern at a frequency f = 30.95 Hz.
Höhman *et al.* 2010, Phys. Rev. Lett. 104 (2010)



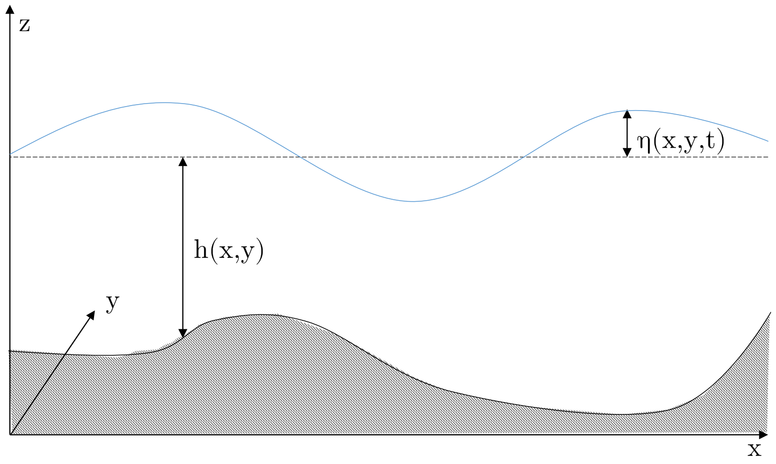
Randomly distributed conical scatterers.
Höhman *et al.* 2010, Phys. Rev. Lett. 104 (2010)

No experimental results for surface water waves so far

Outline

Numerical simulations

Shallow-water equations



1 Linearized shallow-water equation in time domain

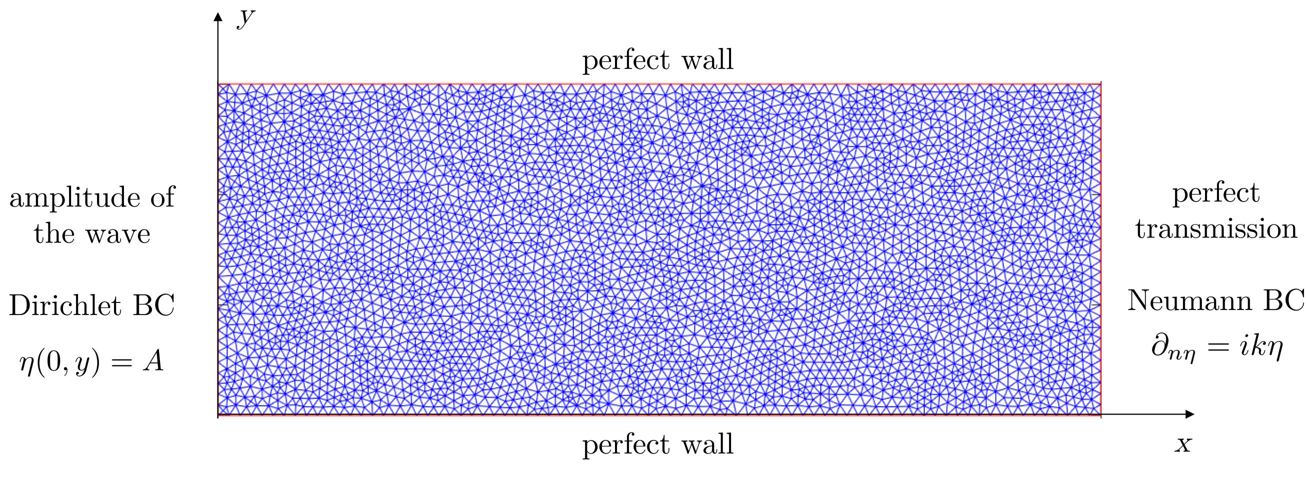
$$\frac{\partial^2}{\partial t^2}\eta + \widetilde{R}\frac{\partial}{\partial t}\eta - \nabla(gh(x,y)\nabla\eta) = 0$$

2 Linearized shallow-water equation in frequency domain (complex solution)

$$\nabla(gh(x,y)\nabla\eta) + (\omega^2 - i\omega\widetilde{R})\eta = 0$$

Final element method

Shallow-water equations



Waves over periodic bathymetry

Bragg's law

1 Shallow-water equation

$$\nabla(h\nabla\eta) + \frac{\omega^2}{g}\eta = 0$$

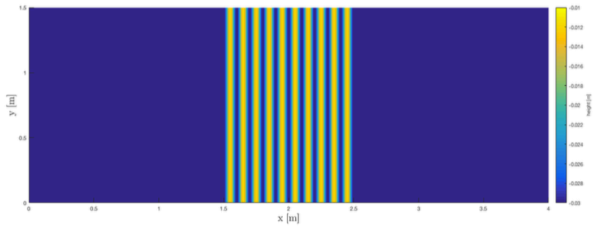
2 Bragg's law

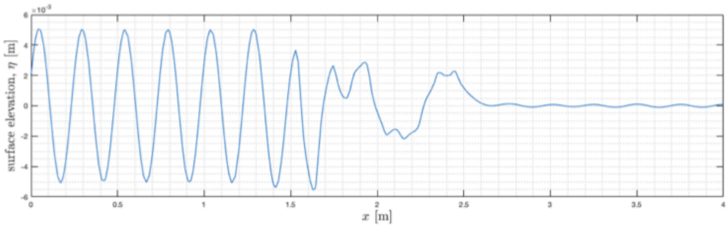
$$\lambda = \frac{2d\sin\theta}{n}$$

3 Dispersion relation

$$\omega^2 = (gk + \frac{\gamma k^3}{\rho})\tanh(kh)$$

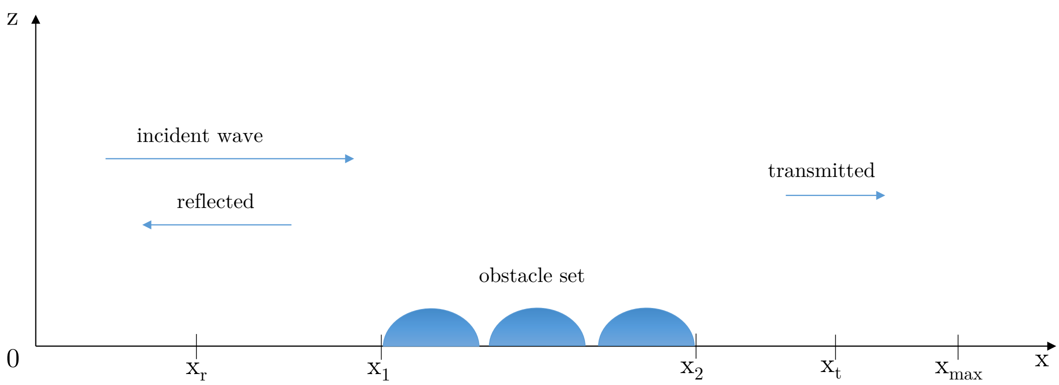
$$f_1 = \frac{\sqrt{gh}}{\lambda} = \frac{\sqrt{gh}}{d} \approx 2.7\text{Hz}$$





Waves over periodic bathymetry

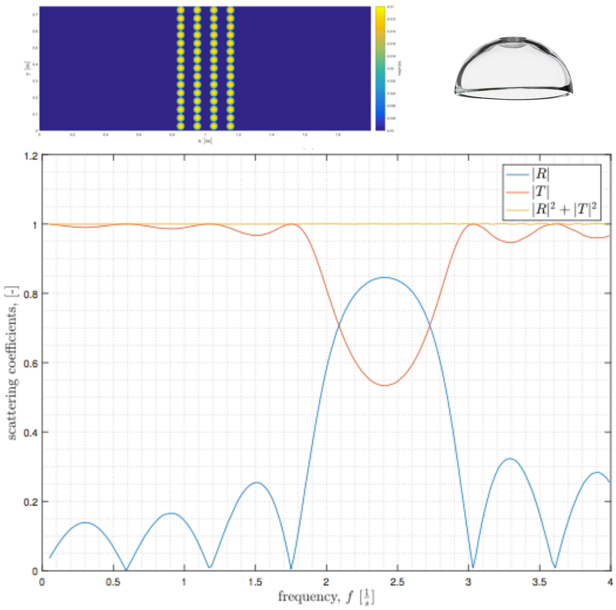
Form of the solution

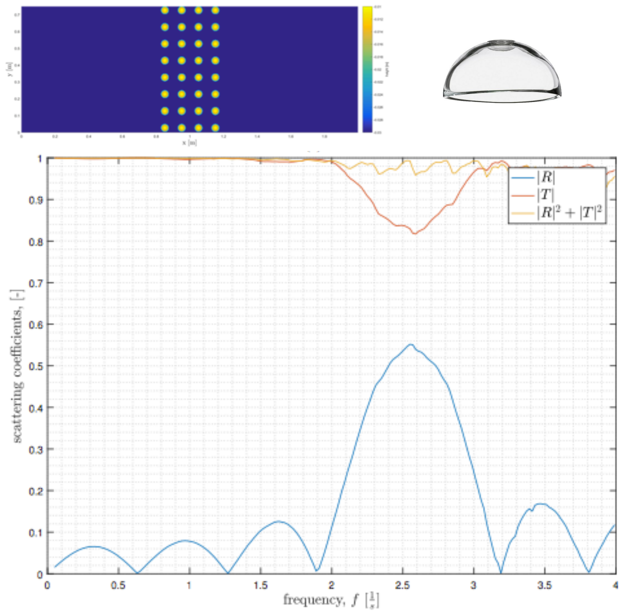


$$\eta(x) = \begin{cases} ae^{-ikx} + Rae^{ikx}, & \text{if } x \in [0, x_1] \\ Tae^{-ikx}, & \text{if } x \in [x_2, x_{max}] \end{cases} \qquad R = -\frac{e^{-ikx_{r_1}} - H_r e^{-ikx_{r_2}}}{e^{ikx_{r_1}} - H_r e^{ikx_{r_2}}} \qquad T = -\frac{e^{-ikx_r} - Re^{ikx_r}}{H_t e^{-ikx_t}}$$

Waves over periodic bathymetry

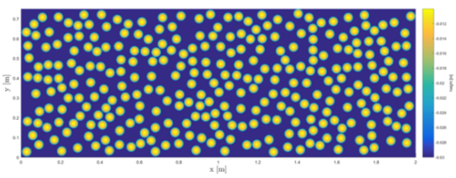
Reflection and transmission coefficients for hemiellipsoid obstacles



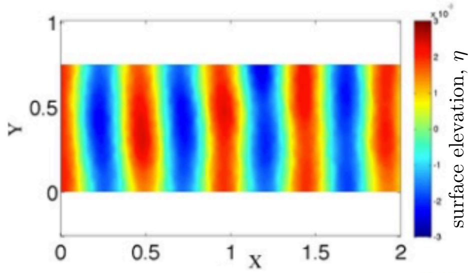


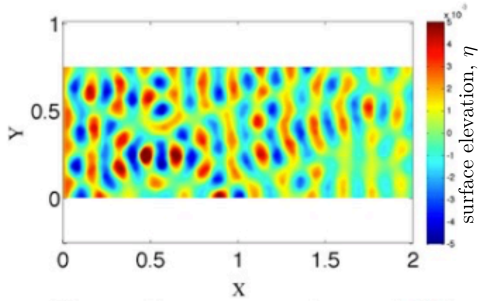
Waves over disordered bathymetry

Branching patterns



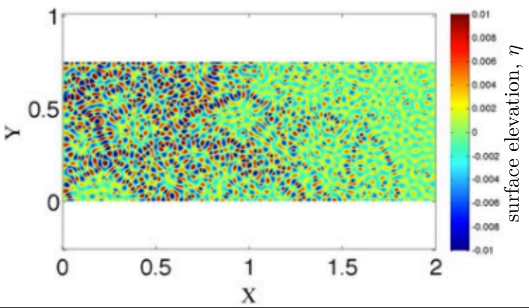
1 Hz

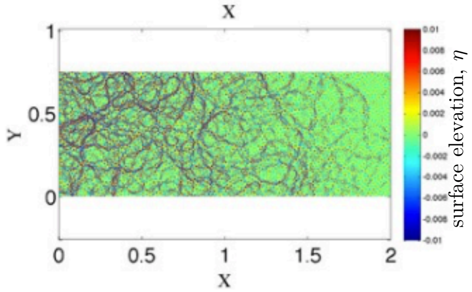




3 Hz

11 Hz

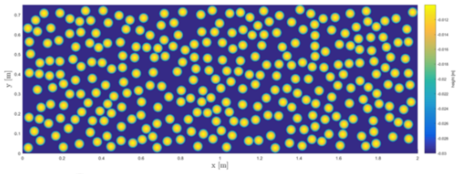


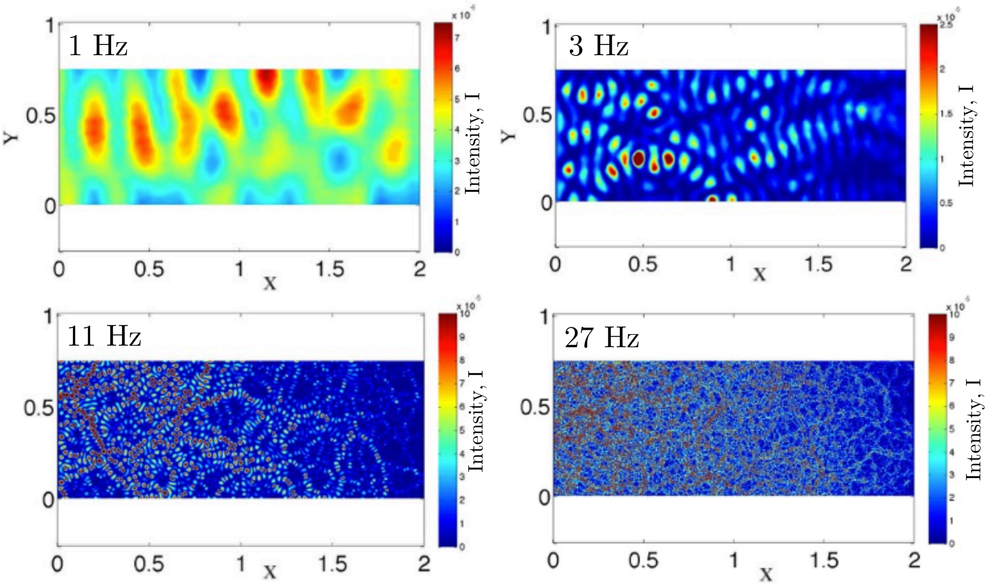


27 Hz

Waves over disordered bathymetry

Intensity maps



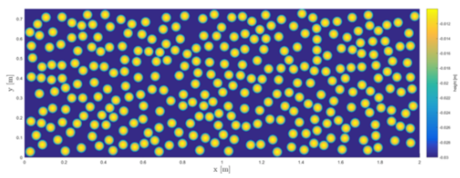


Energy $\propto$ Intensity

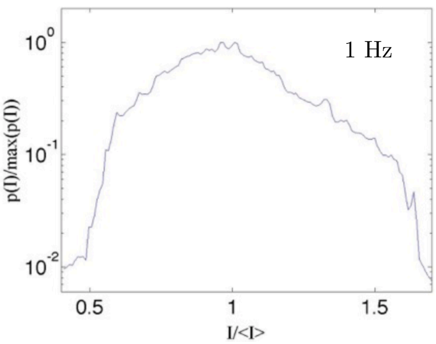
$E \propto I = |\eta|^2 + |\nabla \eta|^2$

Waves over disordered bathymetry

Statistical analysis | Probability density function

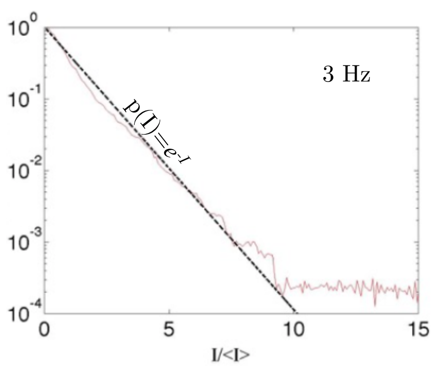


1st regime



Gaussian distribution

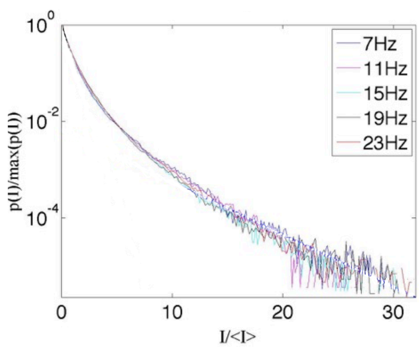
2nd regime



Rayleigh distribution

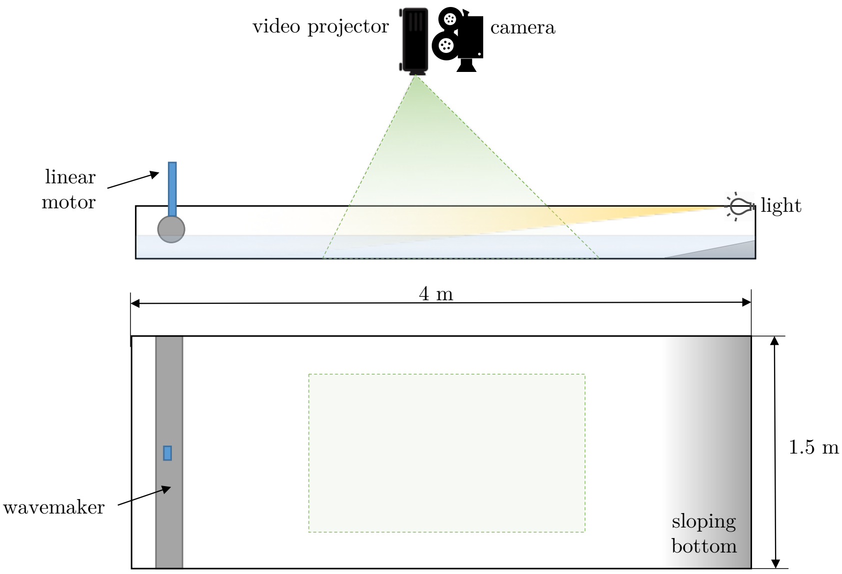
multiple scattering

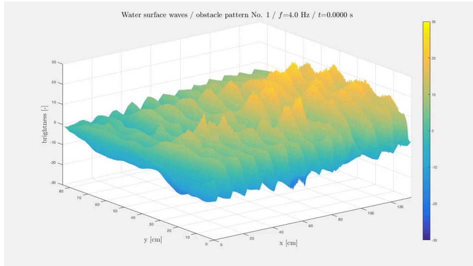
3rd regime

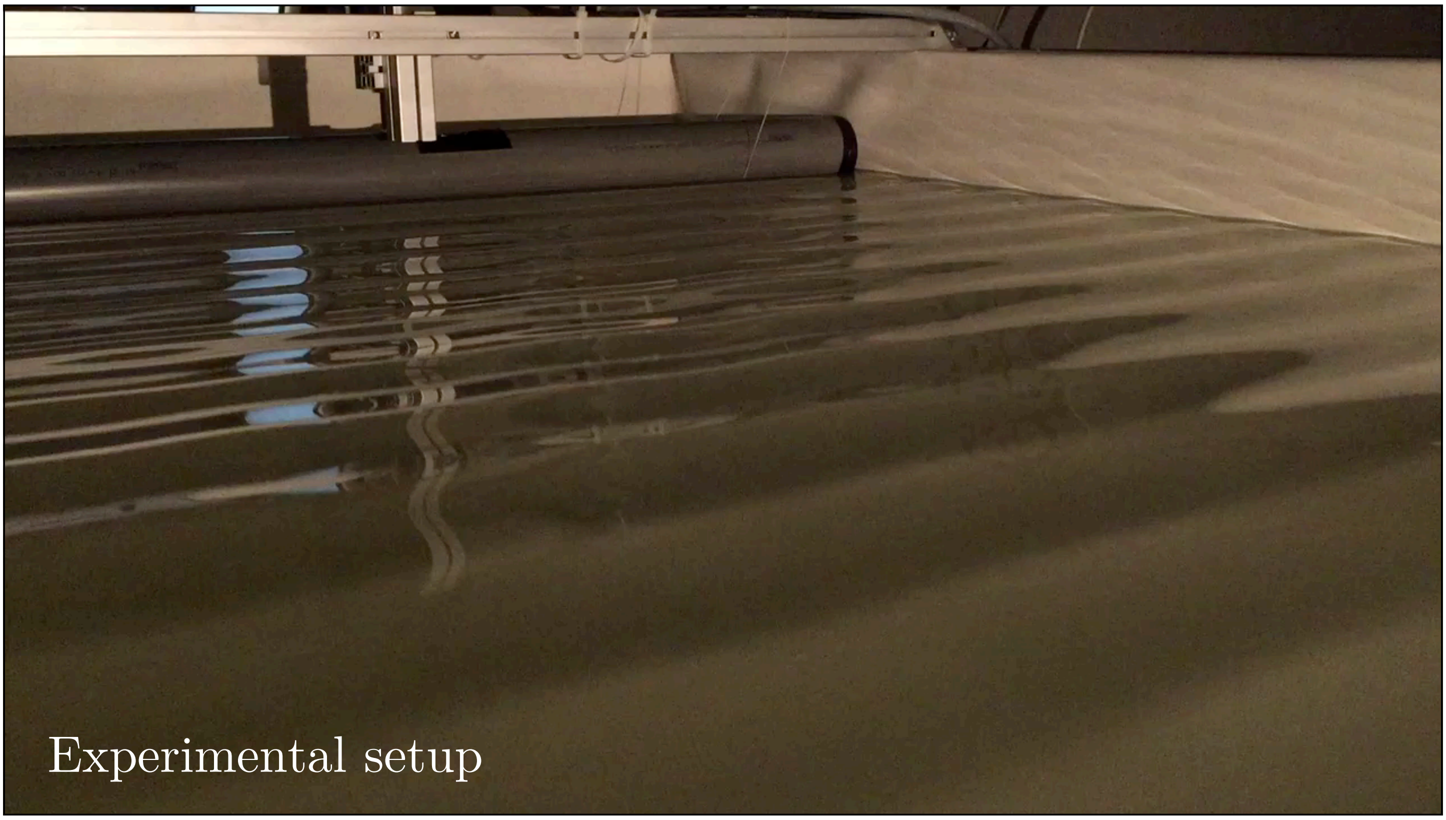


branching patterns

Experimental setup



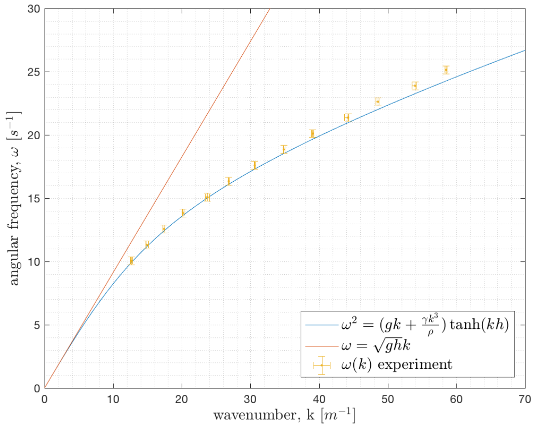




Waves over a flat bottom

Dispersion relation for water surface waes

Wave propagation for a flat bottom and the frequency f = 2.8 Hz

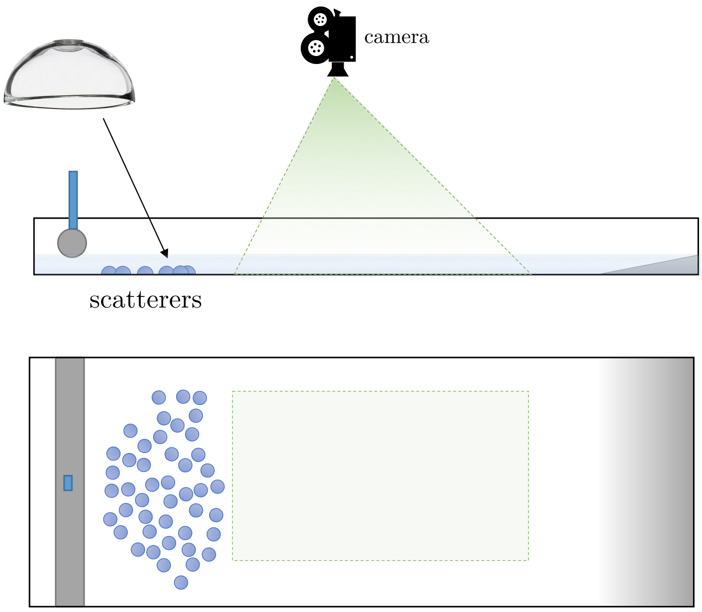


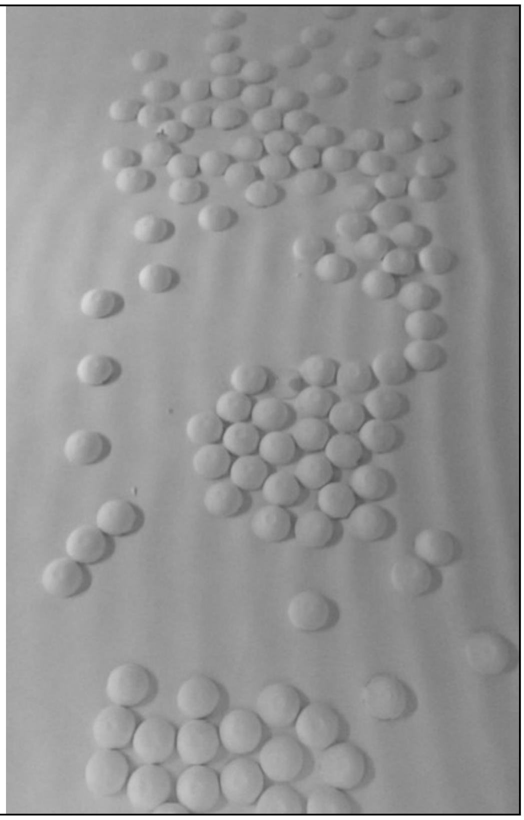
Dispersion relation for water surface waves.

$$\omega^2 = (gk + \frac{\gamma k^3}{\rho})\tanh(kh)$$

Experimental setup

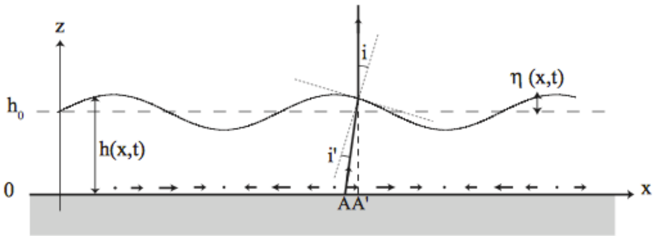
Disordered bathymetry



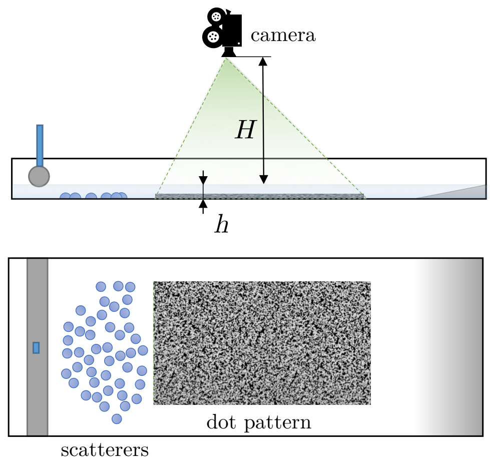


Measurment method

Free-surface synthetic Schlieren



Mesure de la déformation d'une surface libre par analys du déplacement apparent d'un motif aléatoire de points
Moisy *et al.* 18éme Congrés Français de Mécanique (2007)



resolution of ~ 10^{-2} mm

$$\nabla\eta = -\frac{\delta\mathbf{r}}{h^*}, \text{where } \frac{1}{h^*} = \frac{1}{\alpha h} - \frac{1}{H} \longrightarrow \hat{\eta}(x,y,\omega) = \int_{-\infty}^{\infty} \eta(x,y,t)^{-i\omega t}\mathrm{d}t$$

$$\downarrow$$

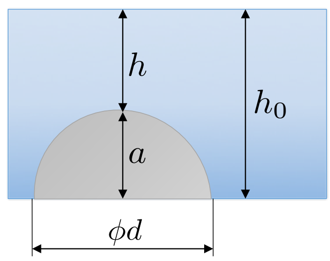
$$I(x,y,\omega_0) = |\hat{\eta}(x,y,\omega_0)|^2$$

$\delta\mathbf{r}$ - optical displacement field

η - free-surface elevation

α - refraction coefficient (0.24 for air-water interface)

Parameters of the system



1 dimensionless wavelength $\lambda^* = \dfrac{\lambda}{d}$

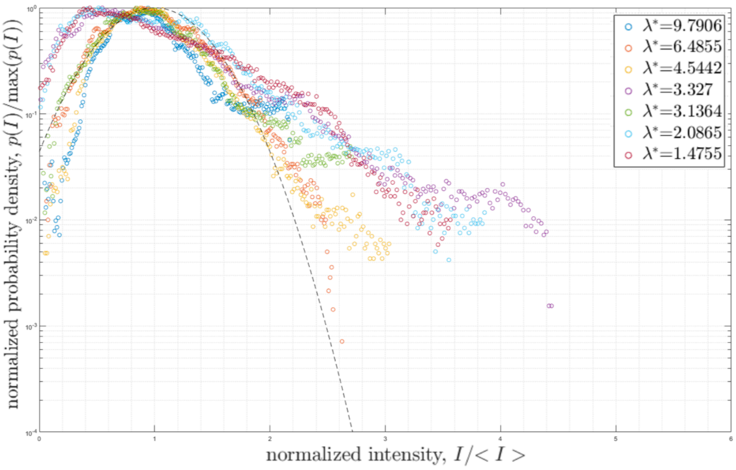
2 strength of the scatterer $k^* = \dfrac{k}{k_0}$

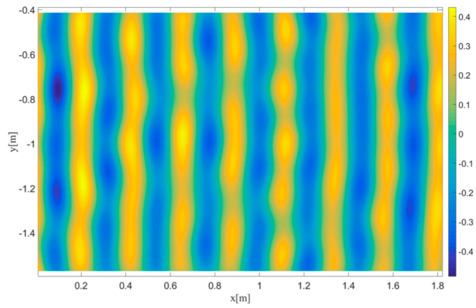
3 density of scatterers $\rho^* = \dfrac{\text{area of scatterers}}{\text{total area}} = \dfrac{N\pi d^2}{4LW}$

4 Ursell number $U = \dfrac{H\lambda^2}{h^3}$

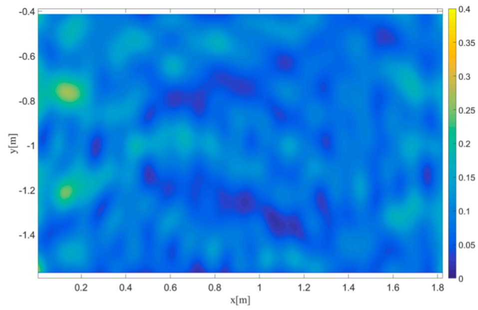
First regime | low frequencies

wavelengths larger than the size of scatterer | $\lambda^*>1$



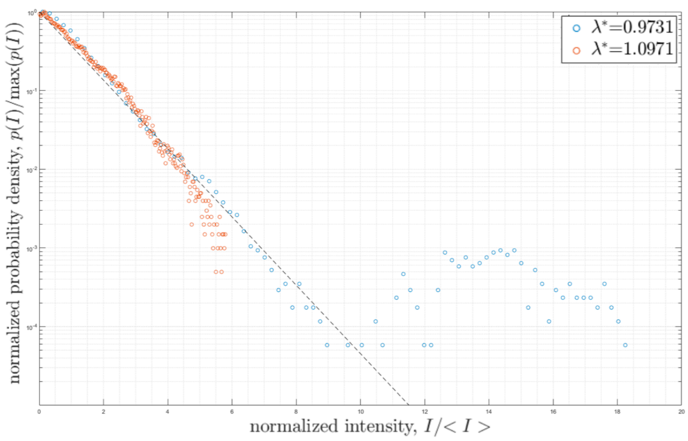


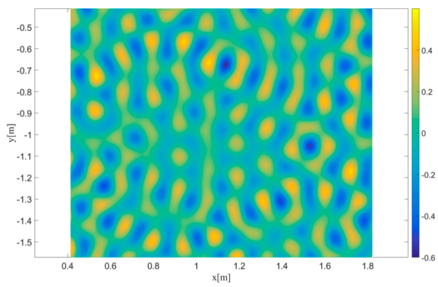
(a)



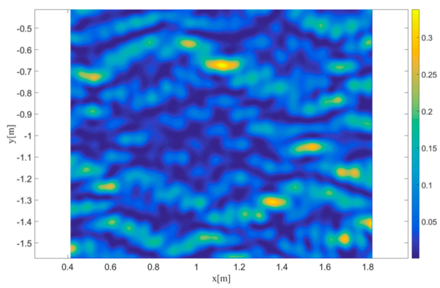
Second regime | intermediate frequencies

wavelengths comparable to the size of scatterer | $\lambda^*\approx 1$



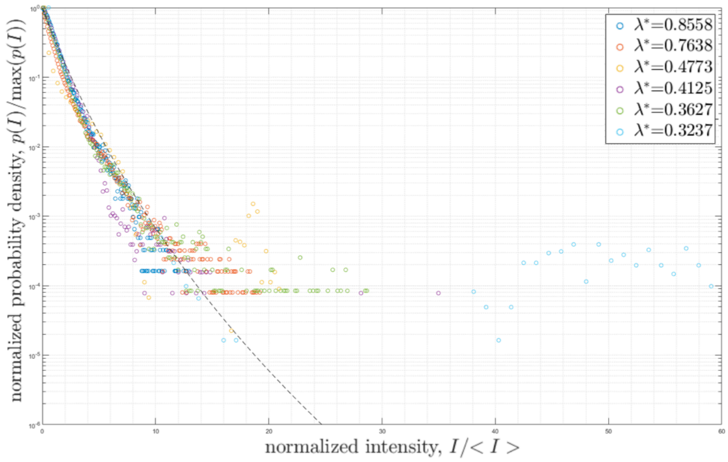


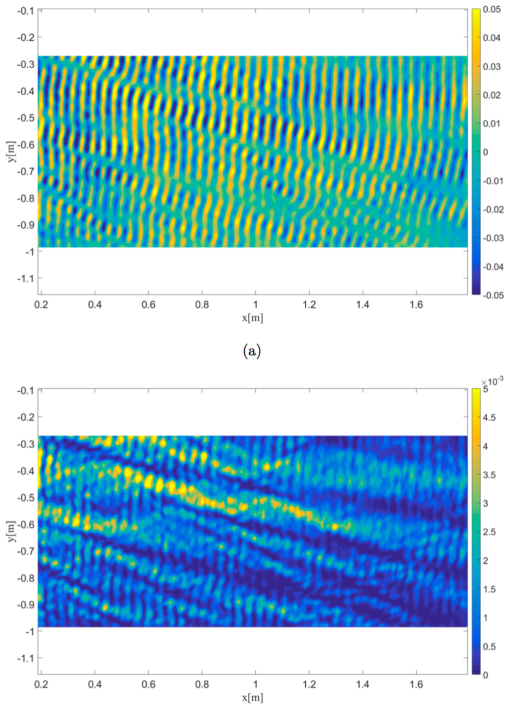
(a)



Third regime | high frequencies

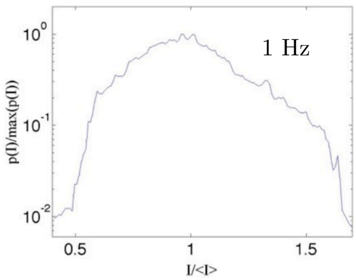
wavelengths smaller than the size of scatterer | $\lambda^*<1$



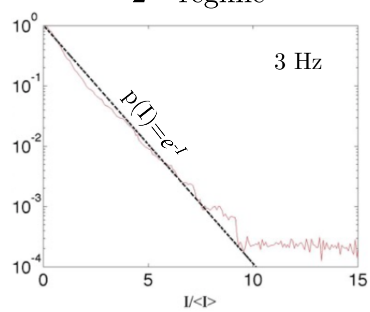


Comparison of numerical and experimental results

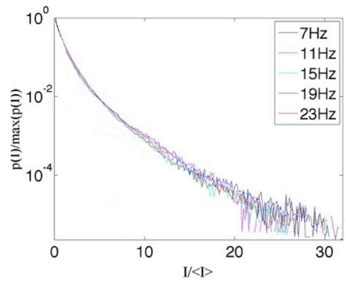
1st regime



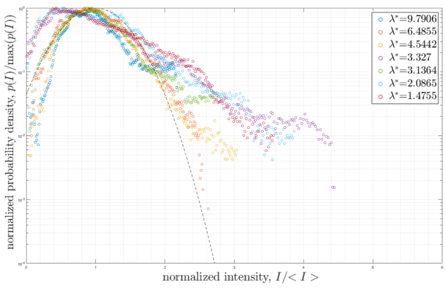
2nd regime



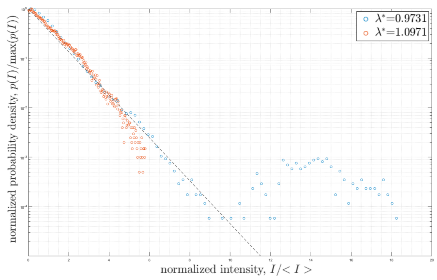
3rd regime



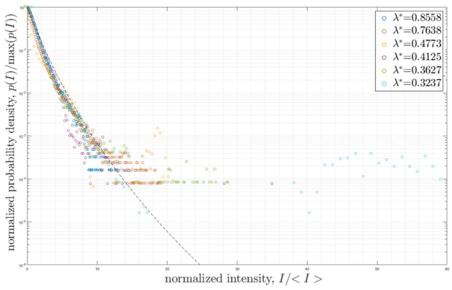
Gaussian distribution $\lambda^*>1$



Rayleigh distribution
multiple scattering $\lambda^*\approx1$



branching patterns $\lambda^*<1$



Summary

- **numerical simulations** have been carried out to obtain suitable parametres for the experiment
  - specified range of frequencies, where branched flow can be observed
- experimental setup **designed and manufactured**
  - implentation of Free-Surface Synthetic Schlieren measurement method
  - construction of **wavemaker** that allowed to acquire needed regime of higher frequencies
- **three regimes** of evolution of branched flow were found numerically and **for the first time confirmed experimentally for water-surface waves**
  - **branched flow** patterns clearly visible for the wavelengths smaller than scatterers