Water waves over variable bathymetry - branched flow in the linear regime

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Branched flow is a phenomenon often found in nature. This phenomenon is of a great interest as it can be observed in many fields of physics. Universal nature of branched flow has been studied through a variety of systems and different orders of magnitude. Such flow is locally amplified and emerges high-energy paths (see Fig.1). It can be observed in areas such as electron flow [1,2], electromagnetic waves [3,4], optics [9-11] or even for water waves [5-8], especially tsunami waves. Tsunami waves in the ocean, which are considered as a shallow-water system, present a branching behaviour that can be of a particular interest and which is difficult to predict. Such an amplification of waves may cause a direct threat to human population around the world. The presented research studies the branched flow in the linear regime for water waves. The main objective is to investigate shallow-water waves in the presence of variable bathymetry and confirm experimentally for the first time the existence of branching structures in water-wave flow. This study can be divided into two parts: the numerical and the experimental one.

The first part concerns an illustration of the phenomenon by numerical simulations. A linearised Shallow Water Equation is implemented in the Finite Element Method solver. A general numerical study was carried out for the waves over periodic and disordered bathymetry. A statistical method for examining the branched flow was presented alongside the definition of three regimes of the branched flow development.

The second part presents the set-up used for the experimental study of interactions between surface waves and random bathymetry. A Free-Surface Synthetic Schlieren method [12,13] is introduced and allows to precisely measure the wave-height field over a large part of the basin. The focusing of energy along branches is shown when the wave passes through the area where the bathymetry is not uniform. Three different regimes of interactions obtained in the numerical part are observed experimentally and a parametric study of the distribution of the intensity of the wave is presented.

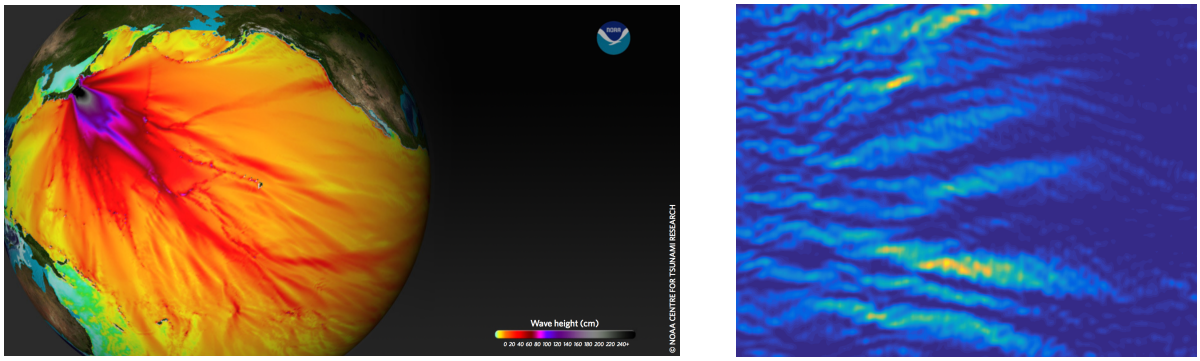

Figure 1: (left) Branched flow seen in the wave energy map produced after the 2011 Sendai earthquake in Japan. High energy path heading for Crescent City in northern California. *National Oceanic and Atmospheric Administration* (2011) [8] (right) Typical intensity map of the water wave flow in the experiment with variable bathymetry.

References

[1] M. A. Topinka, B. J. LeRoy, R. M. Westervelt, S. E. J. Shaw, R. Fleischmann, E. J. Heller, K. D. Maranowski and A. C. Gossard (2001). Coherent branched flow in a two-dimensional electron gas. *Nature (410),* 183–186.

[2] M. P. Jura, M. A. Topinka, L. Urban, A. Yazdani, H. Shtrikman, L. N. Pfeiffer, K. W. West and D. Goldhaber-Gordon (2007). Unexpected features of branched flow through high-mobility two-dimensional electron gases. *Nature Phys. (3),* 841–845.

[3] R. Höhmann, U. Kuhl, H.-J. Stöckmann, L. Kaplan and E.J. Heller (2010). Freak Waves in the Linear Regime: A Microwave Study. *Phys. Rev. Lett. (104).*

[4] J. J. Metzger (2010). Branched Flow and Caustics in Two-Dimensional Random Potentials and Magnetic Fields. *Georg-August-Universität zu Göttingen.*

[5] H. Degueldre, J. J. Metzger, T. Geisel and R. Fleischmann (2016). Random focusing of tsunami waves. *Nature Phys. (12)*, 259–262.

[6] H. Degueldre, J. J. Metzger, E. Schultheis and R. Fleischmann (2017). Channeling of Branched Flow in Weakly Scattering Anisotropic Media. *Phys. Rev. Lett. (118).*

[7] E. J. Heller, L. Kaplan and A. Dahlen (2008). Refraction of a Gaussian Seaway. *J. Geophys. Res. (113).*

[8] E. Heller (2016). Warning from the deep. *Nature Phys. (12).* 824–825

[9] A. Mathis, L. Froehly, S. Toenger, F. Dias, G. Genty and J. M. Dudley (2015). Caustics and Rogue Waves in an Optical Sea. *Scientific Reports (5).*

[10] S. Toenger (2016). Linear and Nonlinear Rogue Waves in Optical Systems. *Université de Franche-Comté.*

[11] M. Mattheakisa, I.J. Pitsiosc, G.P. Tsironisb and S. Tzortzakisc (2016). Extreme events in complex linear and nonlinear photonic media. *Chaos Soliton. Fract. (84)*, 73–80.

[12] F. Moisy, M. Rabaud and K. Salsac (2009). A synthetic Schlieren method for the measurement of the topography of a liquid interface. *Exp Fluids (46)*, 1021–1036.

[13] M. Rabaud, F. Moisy and J. Rolland (2007). Mesure de la déformation d'une surface libre par analyse du déplacement apparent d'un motif aléatoire de points. *18ème Congrès Français de Mécanique, Grenoble, France, August 27-31, 2007. AFM, Maison de la Mécanique.*